Microeconomics C: February 2013 Solutions

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1. (a) Consider the normal-form game below. Do iterated elimination of strictly dominated strategies (explain briefly each step). Which strategies survive?

	E	F	G	H
A	$3,3$	$6,4$	$5,7$	$5,8$
B	$2,5$	$6,1$	$3,8$	$4,9$
C	$1,6$	$8,7$	$6,2$	$6,1$
D	$7,1$	$7,2$	$6,7$	$5,1$

SOLUTION: First B is dominated by D. Then E is dominated by F. Then A is dominated by C. Finally, H is dominated by F and G. Thus the surviving strategies are C and D for player 1 and F and G for player 2.

(b) Consider the following normal-form game (x is a constant):

	L	M	R
U	$4,3$	$2,1$	$5,x$
D	$1,1$	$3,2$	$2,4x$

Find all values of x such that R is a strictly dominated strategy for player 2. For these values of x, find all (pure and mixed) Nash equilibria in the game.

SOLUTION: R is strictly dominated by L when $x < \frac{1}{4}$ and by M when $x < \frac{1}{2}$. Thus R is strictly dominated when $x < \frac{1}{2}$. For these values of x we can simply delete R when we look for Nash equilibria:

	L	M
U	$4,3$	$2,1$
D	$1,1$	$3,2$

The pure strategy NE are (U, L) and (D, M). To find the MNE, let p denote the probability that player 1 plays U and let q denote the probability that player 2 plays L. We get the following conditions:

$$\begin{aligned}4q + 2(1-q) &= q + 3(1-q) \\ 3p + (1-p) &= p + 2(1-p)\end{aligned}$$

From the equations we get $p = \frac{1}{3}$ and $q = \frac{1}{4}$.

2. Consider the following two stage game with two players (1 and 2). In the first stage, player 1 chooses either S (for stop) or G (for go). If he chooses S, the game ends and each player receives a payoff of 3. If he chooses G, the game proceeds to stage two. In stage two, the two players play the following simultaneous game:

		Player 2		
		L	M	R
Player 1	U	$0,0$	$5,1$	$3,0$
	D	$1,3$	$2,2$	$4,2$

(a) Draw a game tree representing the two stage game. Is it a game of perfect or imperfect information? Write down the set of strategies for each player.

SOLUTION: See the game tree on the final page. It is a game of imperfect information because the players choose simultaneously in stage two. The strategies are

$$\begin{aligned} S_1 &= \{GU, GD, SU, SD\} \\ S_2 &= \{L, M, R\} \end{aligned}$$

(b) Find all pure strategy subgame perfect Nash equilibria.

SOLUTION: Since there are two pure strategy NE in the stage two subgame, there are two pure strategy subgame perfect Nash equilibria:

$$(GU, M) \text{ and } (SD, L).$$

(c) Find all pure strategy Nash equilibria. Are they all subgame perfect? Explain.

SOLUTION: Write out the normal-form of the game as a bi-matrix to find all pure strategy NE:

$$(GU, M),\ (SD, L) \text{ and } (SU, L).$$

Only the first two are subgame perfect, see the previous question. The last NE is not subgame perfect because (U, L) is not a NE in the stage two subgame.

3. Two musicians ($i = 1, 2$) are working on a joint song. The effort musician i puts into writing the song is $e_i \geq 0$. The final quality of the song is determined by the effort choices of the musicians in the following way:

$$Q(e_1, e_2) = 2(e_1 + e_2) - Ae_1e_2,$$

where $A > 0$ is a constant. The cost of effort for the musicians are:

$$C_i(e_i) = (e_i)^2 \text{ for each } i = 1, 2.$$

The utility for each musician is equal to the quality of the song minus his cost of effort:

$$U_i(e_1, e_2) = Q(e_1, e_2) - C_i(e_i) \text{ for each } i = 1, 2.$$

(a) Consider the game where the musicians choose their effort levels simultaneously and independently. Find the Nash equilibrium (e_1^*, e_2^*). How do the effort levels depend on A? Give an intuitive explanation.

SOLUTION: To find the NE, first find the best response functions of the musicians and then set up the equilibrium conditions (each musician best responds to the other musician's effort level). Solve the equlibrium conditions to get

$$e_1^* = e_2^* = \frac{2}{2 + A}.$$

The effort level depends negatively on A because a higher A means that the marginal disutility for musician i of putting in more effort is higher (for any level of e_j).

(b) Find the social optimum (e_1^{SO}, e_2^{SO}), i.e., the effort levels that maximize total utility.

SOLUTION: Solve the first order conditions for the problem

$$\max_{e_1, e_2} U_1 + U_2$$

to get

$$e_1^{SO} = e_2^{SO} = \frac{2}{1 + A}.$$

(c) Let $A = 1$. Suppose the game studied in (a) is repeated over an infinite time horizon $t = 1, 2, ..., \infty$. The discount factor of each musician is $\delta \in (0, 1)$. In this infinitely repeated game, specify trigger strategies such that the outcome of each stage is (e_1^{SO}, e_2^{SO}). Find the inequality that must be satisfied for the trigger strategies to constitute a subgame perfect Nash equilibrium. Find the lowest value of δ such that the inequality is satisfied.

SOLUTION: Trigger strategy for musician i:

- If $t = 1$ or if the outcome of all previous stages was (e_1^{SO}, e_2^{SO}), put in the effort e_i^{SO}.
- Otherwise, put in the effort e_i^*.

For musician i, the optimal one shot deviation in the normal phase is $e_i^D = \frac{1}{2}$. The per period payoffs in the normal phase, the punishment phase, and after the one shot deviation are

$$\begin{aligned} U_i^{SO} &= 2(2) - 1 - 1 = 2 \\ U_i^* &= 2(\frac{4}{3}) - \frac{4}{9} - \frac{4}{9} = \frac{16}{9} \\ U_i^D &= 2(\frac{3}{2}) - \frac{1}{2} - \frac{1}{4} = \frac{9}{4} \end{aligned}$$

Thus it is optimal for player i not to deviate in the normal phase precisely if

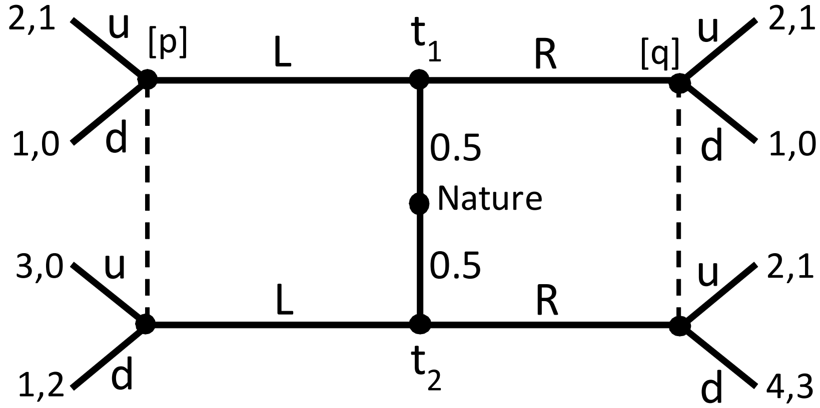
$$\frac{2}{1-\delta} \geq \frac{9}{4} + \frac{16\delta}{9(1-\delta)},$$

which is equivalent to

$$\delta \geq \frac{9}{17}.$$

So if this condition is satisfied, then the trigger strategies constitute a SPNE in the infinitely repeated game.

(a) Find a pooling perfect Bayesian equilibrium in the signaling game below:



SOLUTION: The only pooling PBE are:

$$[(R, R), (d, d), p, q = \frac{1}{2}] \text{ for all } p \leq \frac{2}{3}$$

(b) Two friends, Antonio and Tommy, are bargaining over how to share the 600-gram cake that Antonio's grandmother has baked for them. Antonio's utility from consuming x_A grams of cake is

$$u_A(x_A) = 3x_A.$$

Tommys utility from consuming x_T grams of cake is

$$u_T(x_T) = x_T.$$

If they fail to reach an agreement, Antonio's grandmother will give the cake to one of her neighbors, so that neither of the two friends receives anything.

Represent the situation as a bargaining problem, i.e., draw the sets X and U and mark the disagreement points. Find the Nash bargaining solution.

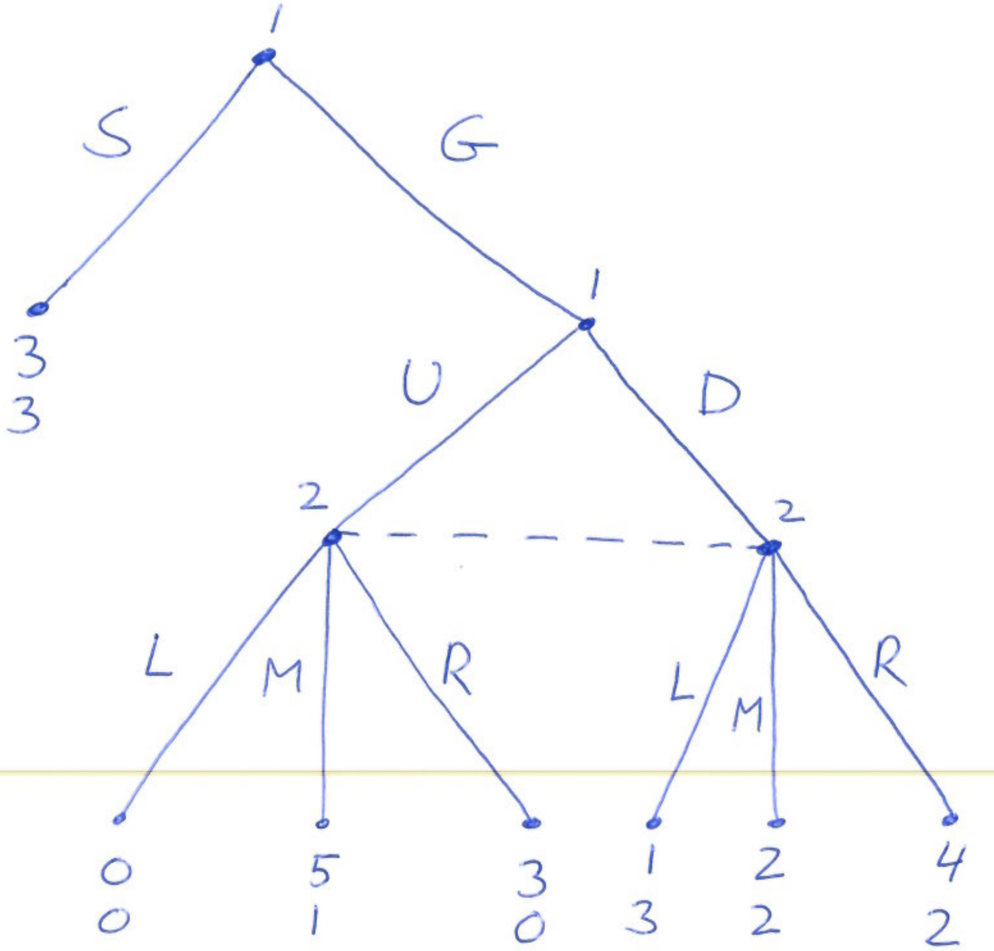
SOLUTION: See the figures on the final page. If we make the following transformation of utilities:

$$\begin{aligned} u'_A &= \frac{1}{3}u_A \\ u'_T &= u_T \end{aligned}$$

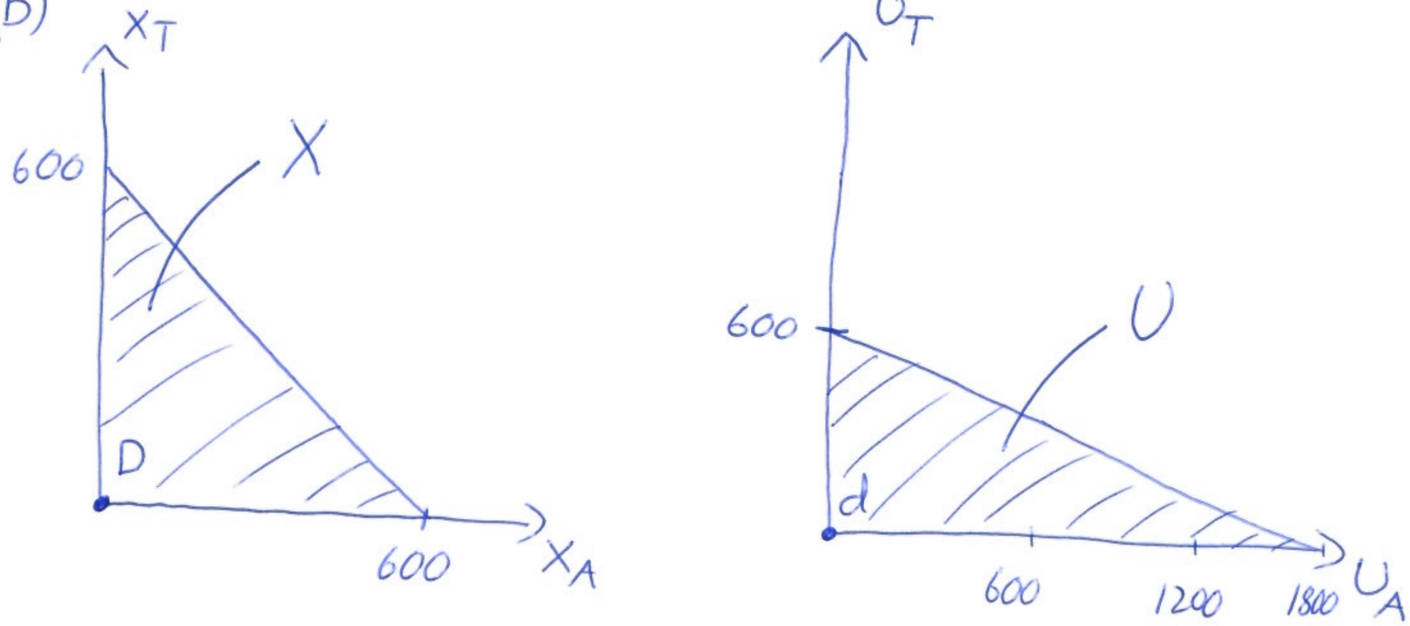
then we get a symmetric problem (U', d'). Then we can use the axioms of symmetry and Pareto efficency to conclude that the solution to the transformed bargaining problem is $(v'_A, v'_T) = (300, 300)$. And then we can use the axiom of invariance to equivalent payoff representations to get that the solution to the original problem is

$$(v_A, v_T) = (3v'_A, v'_T) = (900, 300).$$

2(a)


1
S
G
3
3
1
U
D
2
2
L
M
R
L
M
R
0
0
5
1
3
0
1
3
2
2
4
2

4(b)


x_T
600
X
D
600
x_A
u_T
600
U
d
600
1200
1800
u_A